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Four-dimensional Einstein Yang–Mills de Sitter gravity from eleven dimensions

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Abstract

We obtain $D=4$ de Sitter gravity coupled to $SU(2)$ Yang–Mills gauge fields from an explicit and consistent truncation of $D=11$ supergravity via Kaluza–Klein dimensional reduction on a non-compact space. The "internal" space is a smooth hyperbolic 7-space (H^7) written as a foliation of two 3-spheres, on which the $SU(2)$ Yang–Mills fields reside. The positive cosmological constant is completely fixed by the $SU(2)$ gauge coupling constant. The explicit reduction ansatz enables us to lift any of the $D=4$ solutions to $D=11$. In particular, we obtain dS_2 in M-theory, where the nine-dimensional transverse space is an H^7 bundle over S^2. We also obtain a new smooth embedding of dS_3 in $D=6$ supergravity.

1. Introduction

The embedding of anti-de Sitter (AdS) spacetimes in M-theory and string theories is rather straightforward. In fact, gauged supergravities in diverse dimensions with AdS vacuum have either been shown or are expected to be obtainable from consistent Kaluza–Klein sphere reductions of $D=11$ or $D=10$ supergravities. Notable examples include the simple embedding of $D=4$, $\mathcal{N}=2$, $SU(2)$ Yang–Mills AdS supergravity in $D=11$ [1], the significantly more complicated S^7 [2,3] and S^4 [4–6] reductions of M-theory, and the warped S^4 reduction of massive type IIA theory [7]. Although the S^5 reduction of type IIB theory has yet to be fully established,[3] the reduction of

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[3] The full metric ansatz was conjectured in [8].

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a certain truncation of the theory has been constructed [9,10].

On the other hand, there is less known regarding embedding de Sitter (dS) spacetime in M-theory or string theories, mostly because this is quite a bit more complicated than the case of AdS. With recent experimental evidence suggesting that our universe might be de Sitter [11,12], there is increasing interest in de Sitter gravity in cosmology and the dS/CFT correspondence [13–17]. Thus, it is of importance to obtain the embedding of a non-trivial de Sitter gravity theory in M-theory or string theories.

The first de Sitter solution within the context of an extended supergravity theory was found in [18]. While no-go theorems [19,20] imply that de Sitter spacetime cannot arise from a compactification of a supergravity theory, it can arise from a supergravity theory with a non-compact "internal" space [21]. Explicit embeddings of dS_4 and dS_5 in M-theory and type IIB supergravity, respectively, were obtained in [22]. These arise as ten or eleven-dimensional solutions that have a non-compact hyperbolic internal space.

In this Letter, we obtain four-dimensional de Sitter gravity with $SU(2)$ Yang–Mills gauge fields from a less constrained truncation of $D=11$ supergravity via Kaluza–Klein dimensional reduction on the non-compact space. In this construction, a consistent truncation of the higher-dimensional theory is required in which there are no modes which depend on the internal space. The $SU(2)$ fields arise from modes on the S^3 portions of the internal space. The Yang–Mills gauge coupling constant is completely fixed by the cosmological constant. The kinetic terms for the $SU(2)$ gauge fields have the correct sign, implying that the theory is not merely an analytical continuation of $SU(2)$ AdS supergravity.

This Letter is organized as follows. In Section 2, we rederive the embedding of dS_4 spacetime in M-theory, with the transverse space being an H^7 written as a foliation of two 3-spheres. In Section 3, we propose a reduction ansatz for obtaining $D=4$ $SU(2)$ Yang–Mills de Sitter gravity from $D=11$. We show that the reduction ansatz is indeed consistent with the $D=11$ equations of motion, and hence obtain the Lagrangian for $D=4$ $SU(2)$ Yang–Mills de Sitter gravity. The consistency of the reduction ansatz enables us to lift any $D=4$ solution back to $D=11$. In Section 4, we discuss this in detail. In particular, we embed dS_2 and $(\text{Minkowski})_2$ spacetimes in $D=11$. The internal space is an H^7 bundle over S^2. We also embed a cosmological solution which smoothly interpolates between $dS_2\times S^2$ at the infinite past in the co-moving time to a dS_4-type geometry at the infinite future. In Section 5, we obtain an embedding of dS_3 in $D=6$ supergravity with the transverse space being an H^3 written as a foliation of two circles. We conclude our Letter in Section 6.

2. Embedding dS_4 in $D=11$

We now show how the dS_4 embedding in $D=11$ supergravity found in [22] can be obtained directly from the eleven-dimensional equations of motion. We start with the Lagrangian of the bosonic sector of $D=11$ supergravity, given by

$$\mathcal{L}=R*\mathbb{1}-\frac{1}{2}*F_{(4)}\wedge F_{(4)}-\frac{1}{6}A_{(3)}\wedge F_{(4)}\wedge F_{(4)}, \tag{1}$$

where $F_{(4)}=dA_{(3)}$. We consider the ansatz

$$\begin{aligned}&ds^2=H^2\,ds_4^2+d\rho^2+a^2\,d\Omega_3^2+b^2\,d\tilde{\Omega}_3^2,\\&F_4=q\epsilon_{(4)},\end{aligned} \tag{2}$$

where H, a and b are functions of ρ, ds_4^2 is four-dimensional de Sitter spacetime with cosmological constant $\Lambda=6\lambda^2$, i.e., $R_{\mu\nu}=3\lambda^2 g_{\mu\nu}$, and $\epsilon_{(4)}$ is the corresponding volume-form. $d\Omega_3^2$ and $d\tilde{\Omega}_3^2$ are the metrics of the two unit 3-spheres. The Einstein equations of motion are given by

$$\begin{aligned}&\frac{4\ddot{H}}{H}+\frac{3\ddot{a}}{a}+\frac{3\ddot{b}}{b}=-\frac{q^2}{6H^8},\\&\frac{\ddot{H}}{H}+\frac{3\dot{H}^2}{H^2}+\frac{3\dot{H}}{H}\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right)=\frac{q^2}{3H^8}+\frac{3\lambda^2}{H^2},\\&\frac{\ddot{a}}{a}+\frac{2\dot{a}^2}{a^2}+\frac{3\dot{a}\dot{b}}{ab}+\frac{4\dot{a}\dot{H}}{aH}-\frac{2}{a^2}=-\frac{q^2}{6H^8},\\&\frac{\ddot{b}}{b}+\frac{2\dot{b}^2}{b^2}+\frac{3\dot{a}\dot{b}}{ab}+\frac{4\dot{b}\dot{H}}{bH}-\frac{2}{b^2}=-\frac{q^2}{6H^8},\end{aligned} \tag{3}$$

where a dot represents a derivative with respect to ρ. If the metric ds_4^2 is AdS instead of dS, then one can have a solution with H being a constant. In this case, ρ becomes an angular coordinate with $a\sim\cos\rho$ and $b\sim\sin\rho$, so that the metric becomes the direct product of AdS_4 and S^7, with the seven sphere written as a foliation of two three spheres. Inspired by the sphere reduction ansatz [3], we consider the following redefinition of variables

$$H^2 = \Delta^{\frac{2}{3}}, \qquad a^2 = \Delta^{-\frac{1}{3}}\tilde{a}^2,$$
$$b^2 = \Delta^{-\frac{1}{3}}\tilde{b}^2. \tag{4}$$

Following the analogous relation in the case of $\mathrm{AdS}_4 \times S^7$, we have

$$\tilde{a}^2 = \frac{1}{2}\ell^2(\Delta + 1), \qquad \tilde{b}^2 = \frac{1}{2}\ell^2(\Delta - 1), \tag{5}$$

where ℓ is a constant scale parameter. Substituting (4) and (5) into (3), we find that the constants q and λ must satisfy

$$q^2\ell^2 = 4, \qquad \lambda^2\ell^2 = \frac{4}{3}. \tag{6}$$

Eqs. (3) reduce to a single first-order differential equation

$$\ell^2\Delta^{\frac{2}{3}}\dot{\Delta}^2 - 4\Delta^2 + 4 = 0. \tag{7}$$

Making a coordinate change $d\rho = \ell\Delta^{1/3}\,d\theta$, we can easily solve for Δ, which is given by

$$\Delta = \cosh(2\theta). \tag{8}$$

Thus we have an explicit embedding of dS_4 in $D = 11$ given by

$$\begin{aligned} ds^2 &= \cosh^{\frac{2}{3}}(2\theta)\,ds_4^2 + \ell^2\cosh^{\frac{2}{3}}(2\theta)\,d\theta^2 \\ &\quad + \cosh^{-\frac{1}{3}}(2\theta)\big(\cosh^2\theta\,d\Omega_3^2 + \sinh^2\theta\,d\tilde{\Omega}_3^2\big), \\ F_4 &= \frac{2}{\ell}\epsilon_{(4)}. \end{aligned} \tag{9}$$

This solution was obtained in [22].

Each S^3 in (9) can be replaced by a three-dimensional lens space. A Kaluza–Klein reduction and Hopf T-duality transformation on the fibre coordinate of the two lens spaces yields an embedding of dS_4 in type IIB theory. This procedure was used for the case of AdS solutions in [23].

If we consider (9) as a reduction ansatz from $D = 11$ to $D = 4$, then the resulting four-dimensional theory is Einstein gravity with a positive cosmological constant, and with a corresponding Lagrangian given by

$$e^{-1}\mathcal{L} = R - \frac{8}{\ell^2}. \tag{10}$$

3. Embedding Yang–Mills de Sitter gravity

The metric of the 3-spheres in (9) can be written as

$$\begin{aligned} d\Omega_3^2 &= \frac{1}{4}\big(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\big), \\ d\tilde{\Omega}_3^2 &= \frac{1}{4}\big(\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2 + \tilde{\sigma}_3^2\big), \end{aligned} \tag{11}$$

where σ_i and $\tilde{\sigma}_i$ are $SU(2)$ left-invariant 1-forms satisfying

$$\begin{aligned} d\sigma_i &= -\frac{1}{2}\epsilon_{ijk}\sigma_j \wedge \sigma_k, \\ d\tilde{\sigma}_i &= -\frac{1}{2}\epsilon_{ijk}\tilde{\sigma}_j \wedge \tilde{\sigma}_k. \end{aligned} \tag{12}$$

Thus, we can introduce $SU(2)$ Yang–Mills fields $A^i_{(1)}$ to the vielbein

$$h^i = \sigma_i - gA^i_{(1)}, \qquad \tilde{h}^i = \tilde{\sigma}_i - gA^i_{(1)}. \tag{13}$$

With these preliminaries, we propose the reduction ansatz

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{2}{3}}\,ds_4^2 + g^{-2}\Delta^{\frac{2}{3}}\,d\theta^2 \\ &\quad + \frac{1}{4}g^{-2}\Delta^{-\frac{1}{3}}\Big[c^2\sum_i\big(h^i\big)^2 + s^2\sum_i\big(\tilde{h}^i\big)^2\Big], \end{aligned} \tag{14}$$

$$\begin{aligned} F_{(4)} &= 2g\epsilon_{(4)} \\ &\quad - \frac{1}{4}g^{-2}\Big(sc\,d\theta \wedge h^i \wedge *F^i_{(2)} \\ &\qquad - sc\,d\theta \wedge \tilde{h}^i \wedge *F^i_{(2)} \\ &\qquad - \frac{1}{4}c^2\epsilon_{ijk}h^i \wedge h^j \wedge *F^k_{(2)} \\ &\qquad + \frac{1}{4}s^2\epsilon_{ijk}\tilde{h}^i \wedge \tilde{h}^j \wedge *F^k_{(2)}\Big), \end{aligned} \tag{15}$$

where $c = \cosh\theta$, $s = \sinh\theta$, $\Delta = \cosh(2\theta)$, and $*$ denotes the four-dimensional Hodge dual. Note that we have rewritten the scale parameter ℓ of Section 2 in terms of $g = \ell^{-1}$. The $SU(2)$ Yang–Mills field strengths $F^i_{(2)}$ are given by

$$F^i_{(2)} = dA^i_{(1)} + \frac{1}{2}g\epsilon_{ijk}A^j_{(1)} \wedge A^k_{(1)}. \tag{16}$$

The $D=11$ Hodge dual of the 4-form is given by

$$\begin{aligned}\hat{*}F_{(4)} &= \frac{1}{32}g^{-6}\Delta^{-2}c^3s^3\,d\theta\wedge\epsilon_{(3)}\wedge\tilde{\epsilon}_{(3)}\\ &-\frac{1}{128}g^{-5}\Delta^{-1}c^2s^4\epsilon_{ijk}h^i\wedge h^j\wedge F^k_{(2)}\wedge\tilde{\epsilon}_{(3)}\\ &-\frac{1}{128}g^{-5}\Delta^{-1}s^2c^4\epsilon_{ijk}\tilde{h}^i\wedge\tilde{h}^j\wedge F^i_{(2)}\wedge\epsilon_{(3)}\\ &+\frac{1}{32}g^{-5}cs^3\,d\theta\wedge h^i\wedge F^i_{(2)}\wedge\tilde{\epsilon}_{(3)}\\ &+\frac{1}{32}g^{-5}sc^3\,d\theta\wedge\tilde{h}^i\wedge F^i_{(2)}\wedge\epsilon_{(3)}.\end{aligned}\tag{17}$$

It is straightforward to verify that the Bianchi identity $dF_{(4)}=0$ is satisfied provided that the $SU(2)$ Yang–Mills fields $A^i_{(2)}$ satisfy the lower-dimensional equations of motion

$$D*F^i_{(2)}=0,\tag{18}$$

where the covariant derivative D is defined by $DV^i=dV^i+g\epsilon_{ijk}A^j\wedge V^k$, for any vector V^i. The following identities are useful in verifying the equations of motion

$$\begin{aligned}&DF^i_{(2)}=0,\\ &Dh^i=-\frac{1}{2}\epsilon_{ijk}h^j\wedge h^k-gF^i_{(2)},\\ &D\tilde{h}^i=-\frac{1}{2}\epsilon_{ijk}\tilde{h}^j\wedge\tilde{h}^k-gF^i_{(2)}.\end{aligned}\tag{19}$$

The following formulae are also useful

$$\begin{aligned}&d\big(h^i\wedge *F^i_{(2)}\big)\\ &\quad=Dh^i\wedge *F^i_{(2)}-h^i\wedge D*F^i_{(2)}\\ &\quad=-\frac{1}{2}\epsilon_{ijk}h^j\wedge h^k\wedge *F^i_{(2)}-gF^i_{(2)}\wedge *F^i_{(2)},\\ &\epsilon_{ijk}d\big(h^i\wedge h^j\wedge *F^k_{(2)}\big)\\ &\quad=\epsilon_{ijk}D\big(h^i\wedge h^j\big)\wedge *F^k_{(2)}=0.\end{aligned}\tag{20}$$

The verification of $d\,\hat{*}\,F_{(4)}=\frac{1}{2}F_4\wedge F_{(4)}$ requires the following identity

$$\begin{aligned}&\Delta^{-2}c^3s^3-\frac{1}{2}\big(\Delta^{-1}c^2s^4\big)'+cs^3\ =0,\\ &-\Delta^{-2}c^3s^3-\frac{1}{2}\big(\Delta^{-1}s^2c^4\big)'+sc^3=0.\end{aligned}\tag{21}$$

The evaluation of the $D=11$ Einstein equations of motion are much more complicated, and we have not performed the calculation in full detail. However, we have verified that the equations of motion work for the $U(1)$ subsector of the $SU(2)$ gauge fields. Combining the result, the lower-dimensional equations of motion can be obtained from the Lagrangian

$$e^{-1}\mathcal{L}=R-\frac{1}{4}\big(F^i_{(2)}\big)^2-8g^2.\tag{22}$$

The cosmological constant $8g^2$ is totally fixed by the gauge coupling constant g. Thus, we have obtained four-dimensional Einstein $SU(2)$ Yang–Mills de Sitter gauged gravity from $D=11$ by consistent Kaluza–Klein reduction on a hyperbolic 7-space.

It is worth remarking that $D=11$ supergravity can also give rise to $D=4$ $SU(2)$ AdS supergravity [1]. Also, de Sitter gravity with the wrong sign in the kinetic terms for gauge fields can arise from hyperbolic reduction of $*$ variations of M-theory, type IIB or massive type IIA theories [24,27]. In our reduction of M-theory, however, the sign of the kinetic term for $A^i_{(1)}$ is the right one.

4. Lifting of solutions

The four-dimensional Lagrangian (22) admits a large class of solutions, including multi-center black holes [25–27]. Using our reduction ansatz (14) and (15), all of these solutions can be lifted to eleven dimensions. We will first explicitly lift the case of $dS_2\times S^2$, which is supported by one of the $SU(2)$ gauge fields, e.g., $F^3_{(2)}$. This solution is given by

$$\begin{aligned}&ds_4^2=-d\tau^2+e^{\frac{\tau}{\ell\sqrt{2}}}\,dx^2+4\ell^2\,d\Omega_2^2,\\ &F^3_{(2)}=\frac{1}{2\ell}e^{\frac{\tau}{2\sqrt{2}\ell}}\,d\tau\wedge dx,\qquad *F^3_{(2)}=2\ell\Omega_{(2)},\end{aligned}\tag{23}$$

where $\ell^2=3/(64g^2)$. Note that the role of $F^3_{(2)}$ and $*F^3_{(2)}$ are interchangeable, giving rise to either the electric or magnetic solutions, with the metric unchanged. Lifting this solution back to $D=11$ yields a smooth and regular embedding of dS_2 given by

$$\begin{aligned}
ds_{11}^2 &= \Delta^{\frac{2}{3}}\big(-d\tau^2 + e^{\frac{\tau}{\ell\sqrt{2}}}dx^2 + 4\ell^2\, d\Omega_2^2\big) \\
&\quad + g^{-2}\Delta^{\frac{2}{3}}\, d\theta^2 + \frac{1}{4}g^{-2}\Delta^{-\frac{1}{3}} \\
&\quad \times \big[c^2\big(d\omega_2^2 + \big(\sigma_3 - \sqrt{2}\, g e^{\frac{\tau}{2\sqrt{2}\ell}}\, dx\big)^2\big) \\
&\qquad + s^2\big(d\tilde{\omega}_2^2 + \big(\tilde{\sigma}_3 - \sqrt{2}\, g e^{\frac{\tau}{2\sqrt{2}\ell}}\, dx\big)^2\big)\big], \\
F_{(4)} &= 8g\ell^2 e^{\frac{\tau}{2\sqrt{2}\ell}}\, d\tau\wedge dx\wedge\Omega_{(2)} \\
&\quad - \frac{1}{2}g^{-2}\ell\bigg(sc\, d\theta\wedge(\sigma_3 - \tilde{\sigma}_3) \\
&\quad - \frac{1}{2}c^2\omega_{(2)}\wedge + \frac{1}{2}s^2\tilde{\omega}_{(2)}\bigg)\wedge\Omega_{(2)}.
\end{aligned} \tag{24}$$

We explicitly verified that the above solution satisfies the equations of motion of $D = 11$ supergravity, which serves as a double check of our reduction ansatz (14) and (15). In this smooth embedding of dS_2 in M-theory, the metric can be viewed as a rotating brane. Of course, if we interchange the role of $F_{(2)}^3$ and $*F_{(2)}^3$ in (23), then the transverse space is a nine-dimensional non-compact space which can be viewed as an H^7 bundle over S^2.

We will now consider a regular cosmological solution of (22) given by

$$\begin{aligned}
ds_4^2 &= H^2\big(-f^{-1}dt^2 + f\, dx^2 + t^2\, d\Omega_2^2\big), \\
F_{(2)}^3 &= \frac{2\ell}{(tH)^2}dt\wedge dx, \qquad *F_{(2)}^3 = 2\ell\Omega_{(2)}, \\
H &= 1 + \frac{\ell}{t}, \qquad f = \frac{4}{3}g^2t^2H^4 - 1.
\end{aligned} \tag{25}$$

This solution is, in fact, nothing but the BPS AdS Reissner–Nordstrøm black hole with $g \to \mathrm{i}g$ [28]. When $g^2\ell^2 = \frac{3}{64}$, the solution interpolates between $dS_2\times S^2$ at the infinite past in the co-moving time to a dS_4-type geometry at the infinite future with the boundary of $S^2\times S^1$ [28]. It is straightforward to lift the solution back to $D = 11$ and obtain a regular cosmological solution in M-theory. The corresponding metric is given by

$$\begin{aligned}
ds_{11}^2 &= \Delta^{\frac{2}{3}}H^2\big(-f^{-1}dt^2 + f\, dx^2 + t^2\, d\Omega_2^2\big) \\
&\quad + g^{-2}\Delta^{\frac{2}{3}}\, d\theta^2 \\
&\quad + \frac{1}{4}g^{-2}\Delta^{\frac{-1}{3}}\bigg[c^2\bigg(d\omega_2^2 + \bigg(\sigma_3 - \frac{2g}{H}\, dx\bigg)^2\bigg) \\
&\quad + s^2\bigg(d\tilde{\omega}_2^2 + \bigg(\tilde{\sigma}_3 - \frac{2g}{H}\, dx\bigg)^2\bigg)\bigg].
\end{aligned} \tag{26}$$

As a final example, we will lift $(\mathrm{Minkowski})_2\times S^2$ to eleven dimensions. This four-dimensional solution is given by

$$\begin{aligned}
ds_4^2 &= -d\tau^2 + dx^2 + \frac{1}{8g^2}\, d\Omega_2^2, \\
F_{(2)}^3 &= 4g\, d\tau\wedge dx, \qquad *F_{(2)}^3 = \frac{1}{2g}\Omega_{(2)}.
\end{aligned} \tag{27}$$

Lifting this solution back to $D = 11$ yields a smooth and regular embedding of M_2, whose metric is given by

$$\begin{aligned}
ds_{11}^2 &= \Delta^{\frac{2}{3}}\bigg(-d\tau^2 + dx^2 + \frac{1}{8g^2}\, d\Omega_2^2\bigg) \\
&\quad + g^{-2}\Delta^{\frac{2}{3}}d\theta^2 + \frac{1}{4}g^{-2}\Delta^{-\frac{1}{3}} \\
&\quad \times \big[c^2\big(d\omega_2^2 + \big(\sigma_3 - 4g^2\, dx\big)^2\big) \\
&\qquad + s^2\big(d\tilde{\omega}_2^2 + \big(\tilde{\sigma}_3 - 4g^2\, dx\big)^2\big)\big].
\end{aligned} \tag{28}$$

As in the previous examples, one can interchange the role of $F_{(2)}^3$ and $*F_{(2)}^3$ in (27).

5. Further embeddings of dS spacetime

In [22], there are further embeddings of dS spacetime in M-theory or type IIB supergravities. In our example of a dS_4 embedding in M-theory, the H^7 is a foliation of $S^3\times S^3$. The embeddings of dS_4 in M-theory with a squashed H^7 being a foliation of $S^2\times S^4$ and dS_5 in type IIB theory with the H^5 being a foliation of $S^2\times S^2$ were obtained in [22]. In this section, we obtain an new embedding of dS_3 in $D = 6$ supergravity. The relevant Lagrangian is given by

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12}e^{\sqrt{2}\phi}F_{(3)}^2. \tag{29}$$

The solution is given by

$$\begin{aligned}
ds_6^2 &= \Delta\, ds_3^2 + g^{-2}\Delta\, d\theta^2 \\
&\quad + g^{-2}\Delta^{-1}\big(c^2\, d\phi_1^2 + s^2\, d\phi_2^2\big), \\
F_{(3)} &= 2g\big(\epsilon_{(3)} + *\epsilon_{(3)}\big), \qquad \phi = 0,
\end{aligned} \tag{30}$$

where ds_3^2 is dS_3 spacetime with cosmological constant $\Lambda = 2g^2$. Note that the transverse space is an H^3

written as a foliation of two circles. This theory can be reduced to pure de Sitter gravity in three dimensions, with the corresponding Lagrangian given by

$$e^{-1}\mathcal{L}_3 = R - 2g^2. \qquad (31)$$

6. Conclusions

We have obtained $D = 4$ de Sitter gravity coupled to $SU(2)$ Yang–Mills fields from a consistent Kaluza–Klein reduction and truncation of $D = 11$ supergravity on a hyperbolic 7-space. The hyperbolic space is written as a foliation of two 3-spheres, on which the $SU(2)$ fields are embedded. Unlike in the case of $*$ theories, the kinetic terms of the gauge fields have the correct sign. The four-dimensional cosmological constant is completely fixed by the gauge coupling constant. Although our reduction procedure fits within the general pattern of non-compact gaugings and their higher-dimensional origins, described in [21], our result provides the first explicit embedding of a non-trivial de Sitter gauge gravity in M-theory.

The reduction ansatz enables us to lift any solution of the four-dimensional theory to eleven dimensions. We discuss the embeddings of smooth cosmological solutions. In particular, we obtain the embeddings of dS_2 and M_2 in M-theory, as well as that of a cosmological solution which smoothly interpolates between $dS_2 \times S^2$ at the infinite past in the co-moving time to a dS_4-type geometry at the infinite future. We also obtain a new embedding of dS_3 in $D = 6$ supergravity. Our results provide important tools with which to study the dS/CFT correspondence from the point of view of string and M-theory.

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